# Design of an Active 2nd-Order Polyphase Filter for Image Rejection in Subsampling Receivers

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***Abstract*—Receivers that work with real subsampling, have the disadvantage that they can not transform a bandpass signal directly to baseband. By using complex subsampling, direct conversion into the baseband is made possible. For this purpose, the bandpass signal must first be converted into a complex or analytic signal. This signal, consisting of real and imaginary parts, is filtered by a polyphase filter. This type of filter allows the transition of positive frequencies and filtering the corresponding negative frequencies.**

**In order to ensure the requirements of the receiver, a high image rejection between negative and positive frequencies must be achieved. This can be done by cascading several first-order filters.**

**To obtain a lower cost alternative of higher order filters, second order filters can be cascaded. This kind of implementation requires only half as many integrators. The disadvantages of second-order polyphase filters are the more complex design and the higher sensitivity to interference and component tolerances. For this reason, this paper focuses specifically on the design and verification of the second-order polyphase filters on a printed-circuit-board (PCB).**

***Index Terms*—Active Polyphase Filter, Complex Subsampling, Image Rejection, Direct Baseband Conversion, Gingell Passive Polyphase Filter.**

## I. INTRODUCTION

In order to be able to transform a band-limited signal directly into the baseband and at the same time use the smallest possible sampling frequency, it is necessary to filter one side of the frequency spectrum so that only positive or negative frequencies are present. In this technique – called image rejection – the real and imaginary parts of a signal are sampled separately by two analog-to-digital converters (ADCs). This complex or analytical signal is filtered using a polyphase filter. Due to the complex transfer function, this filter makes it possible to pass a band of positive frequencies and suppress the associated negative frequencies, or vice versa. Most digital communication systems require an image rejection of at least 35 dB. However, this is only the case if the wanted and the mirrored signal are similar. If this is not the case, at least 55 dB should be achieved [5], [8].

Implementations of passive polyphase filters and first-order active polyphase filters are described in [8], [9] and [13]. The disadvantage of these filter stages is that they have to be cascaded in order to achieve the required image rejection. This results in a higher signal loss, higher sensitivity to mismatches and higher energy consumption. By implementing a second-order active polyphase filter, it is possible to halve the number of filter stages. The active second-order filters described in this paper is based on calculations and simulations from [3].

### A. Complex subsampling

The output of an image rejection polyphase filter is an analytical signal $X_+(t) = X_I(t) + jX_Q(t)$ with in-phase $X_I$ and quadrature phase $X_Q$. The generated complex signal has only, either positive or negative frequency components. Figure 1 shows the spectrum of the generated complex or analytical signal $x_+(t)$.

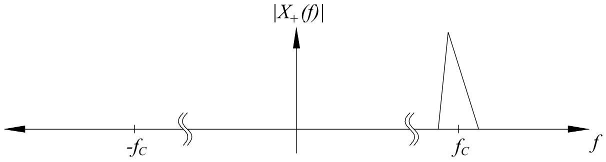


Fig. 1. Spectrum of an analytic signals $x_+(t)$ [1].

The two real signals that make up the complex signal can now be sampled at a sampling frequency lower than $f_S = 2 \cdot B$. This is called complex subsampling [4]. Since no negative frequency components can be superimposed with the positive ones, the condition in (1) is sufficient for the sampling frequency $f_S$.

$$f_S \geq B \tag{1}$$

If this condition is met, no aliasing occurs. Furthermore, if the following equation (2) matches, the signal can be transformed directly into the baseband.

$$f_C = nf_S \ ; \quad n \in \mathbb{N} \tag{2}$$

The resulting spectrum is shown in Figure 2. The signal was sampled with the minimum sampling frequency $f_S = B$. Since there are no image frequencies, there is no overlay [2].

## II. THEORETICAL PRINCIPLES

Polyphase filters can be differentiated into passive and active filters. As the name suggests, a passive polyphase filter consists of purely passive components, whereas active filters also have active components.

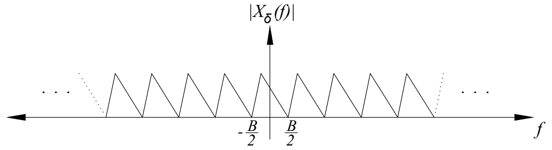


Fig. 2. Spectrum of a signal sampled at the minimum sampling frequency $x_\delta(t)$ [1].

### A. Passive Polyphase Filter

A passive polyphase filter has no active devices but consists of a network of resistors and capacitors. Figure 3 shows one possibility of a passive polyphase filter stage according to Gingell [6]. Firstly, the in-phase and quadrature phase are differently wired. Moreover, the output waveforms of the I/Q phases are phase shifted by 90° within a suitable frequency range.

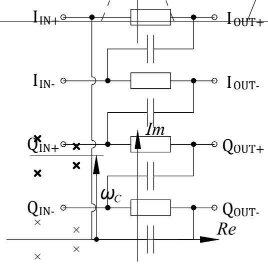
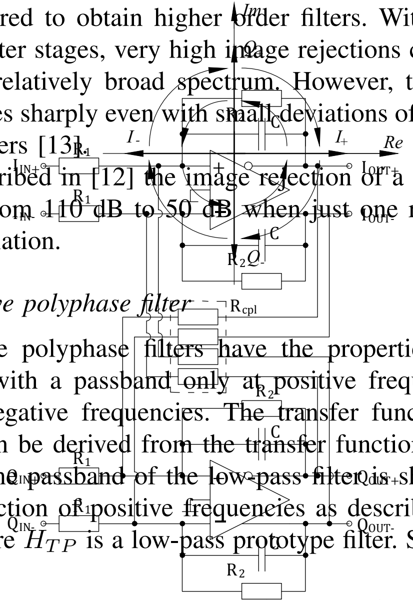


Fig. 3. Structure of a 4-phase first-order passive polyphase filter with cyclic coupling [9].

This single filter stage can be connected in series as often as required to obtain higher order filters. With ideal higher-order filter stages, very high image rejections can be achieved over a relatively broad spectrum. However, this suppression decreases sharply even with small deviations of the component parameters [13].
As described in [12] the image rejection of a fifth-order filter drops from 110 dB to 50 dB when just one resistance has a 1% deviation.



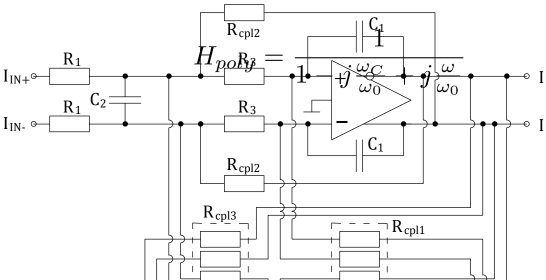
### B. Active polyphase filter

Active polyphase filters have the properties of bandpass filters, with a passband only at positive frequencies or vice versa negative frequencies. The transfer function of such a filter can be derived from the transfer function of a low-pass filter. The passband of the low-pass filter is shifted by $\omega_C$ in the direction of positive frequencies as described in equation (3) where $H_{TP}$ is a low-pass prototype filter. See e.g. [7], [9].

$$H_{poly} = H_{TP}(j\omega - j\omega_C) \tag{3}$$

Equation (4) shows the transfer function of a polyphase filter derived from the transfer function of a first-order low-pass filter.

$$H_{poly} = \frac{1}{1 - j\frac{\omega_C}{\omega_0} + j\frac{\omega}{\omega_0}} \tag{4}$$



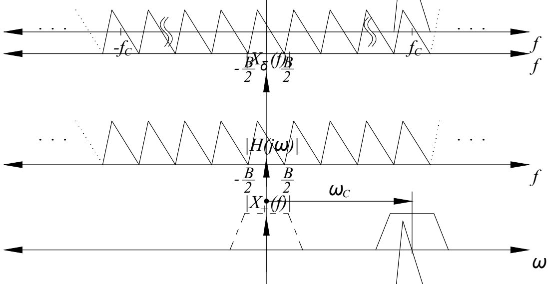


Fig. 4. Shift of the passband towards positive frequencies [7].

Figure 4 depicts the shift of the passband from a low-pass prototype filter towards positive frequencies.

In addition to the shift of the frequency range, the poles and zeros on the complex plane, along the imaginary axis, are also shifted by the value $\omega_C$ (Figure 5). This results in an asymmetric behaviour along the imaginary axis which characterizes a complex filter.

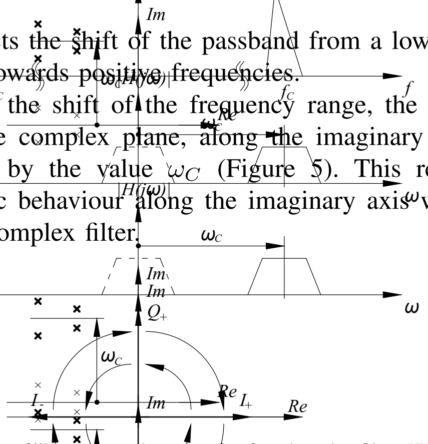


Fig. 5. Shifting of the pole and zeros of a fourth-order filter [7].

The coupling within asymmetric polyphase filters is arranged in a cyclic way. Figure 7 depicts such a first-order filter with cyclic coupling of the two differential low-pass filters by $R_{cpl}$. Figure 6 shows how this is done in the complex plane [5].

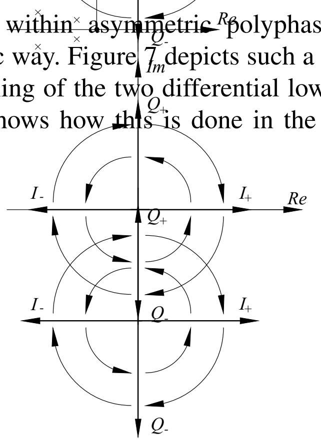


Fig. 6. Cyclic coupling in the complex plane [5].

Due to the cyclical arrangement of the elements in the circuit, it is possible to determine the resulting transfer functions of such a system. Second-order asymmetric polyphase filters use both cyclic directions for their two coupled groups of elements.

The cyclic coupling of the fed back outputs forms a circular matrix. This circular matrix has four eigenvectors, that are the column vectors of the Discrete Fourier Transformation (DFT). Equation (5) shows these eigenvectors $z$ [5]. The parameters were calculated using a MuPAD script from [3].

$$z = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{pmatrix} \tag{5}$$

The second column vector represents the eigenvector of the wanted signal and shall be amplified by the filter stage, whereas the fourth column vector represents the image signal which shall be filtered. The first and third column vectors are irrelevant for the filter design.

*1) First-order filter:* Figure 7 represents the realization of a first-order active polyphase filter. It has a single pole, which is shifted by the value $\omega_0 = 2\pi f_0$ along the imaginary axis. The component parameters of the resistors and capacitors can be calculated using the following equations (6), where $A_0$ is the gain, $f_0$ describes the shift frequency and $f_C$ the cutoff frequency of the corresponding baseband filter.

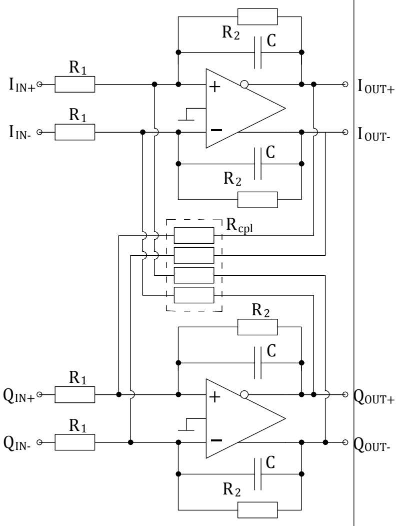


Fig. 7. Realization of a first-order polyphase filter [8].

$$R_1 = \frac{R_2}{A_0}\ ; \quad R_2 = \frac{1}{C 2\pi f_0}\ ; \quad R_{cpl} = R_2 \frac{f_0}{f_C} \qquad (6)$$

*2) Second-order filter:* Also, the second-order polyphase filter can be derived from a corresponding second-order low-pass filter. This filter has two feedback branches and is able to realize conjugated complex pole pairs. Thus, the design of a filter with e.g. Butterworth or Chebyshev characteristic is possible [11]. Figure 8 shows the structure of such a multiple feedback low-pass filter. For a polyphase realization, the multiple feedback filter is realized by differential signaling too.

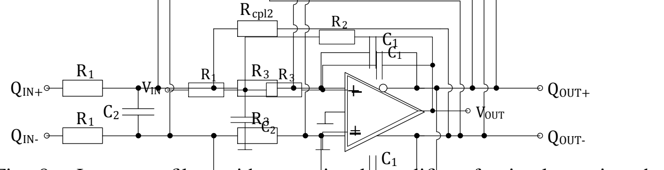


Fig. 8. Low-pass filter with operational amplifiers for implementing the second-order polyphase filter [10].

Like the first-order polyphase filter, this filter requires only two differential amplifiers but has a theoretical slope of −40 dB per decade. This is twice the value of the first-order filter. Figure 9 shows the structure of a second-order polyphase filter derived from the low-pass filter previously described. This design by [3] is slightly different from the design of the low-pass filter. Here, $R_{cpl2}$ is not fed back from the equivalent output, but from the opposite one. This is necessary because otherwise the interpretation of the filter parameters is often not possible. If the difference between cutoff frequency and shift frequency is too high, very small or even negative parameter values would be needed in this case [5]. As described, a MuPAD script from [3] was used to calculate the components.

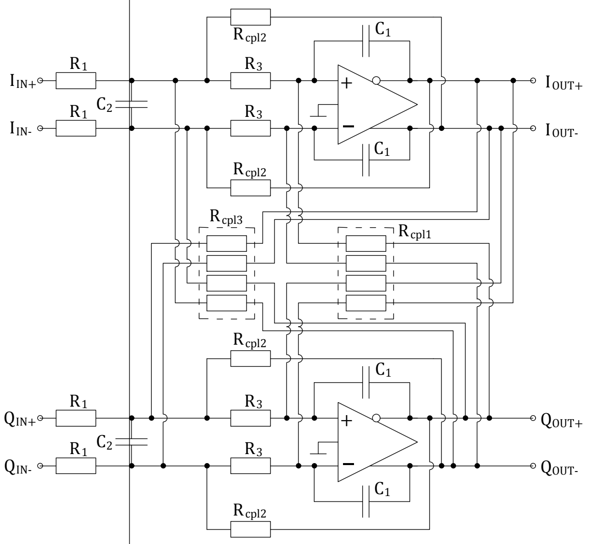


Fig. 9. Realization of a second-order polyphase filter.

## III. RESULTS

### A. Layout

For the layout design, particular attention was paid to the lengths of the feedback paths and the symmetrical arrangement of the components to reduce I/Q interference. The AD8132 from Analog Devices was chosen as the differential amplifier. The resistances were chosen with a maximum tolerance of 2% and the NP0 capacitors with 5%. The two-layer PCB, with a signal wire width of 6 mils and a via diameter of 12 mils, was manufactured by PCB-Pool. Figure 10 shows the finished PCB.

Fig. 10. PCB of the second-order polyphase filter.

### B. Measurement setup

With a signal generator sinusoidal voltages of different frequencies were applied to the inputs of the filter. For positive frequencies, the phase between I and Q was +90° and for negative frequencies −90°. The amplitude and phase of the output voltage were measured with an oscilloscope. The determined points for magnitude and phase were transferred to a Bode diagram. The measurement setup is shown in Figure 11 [12].

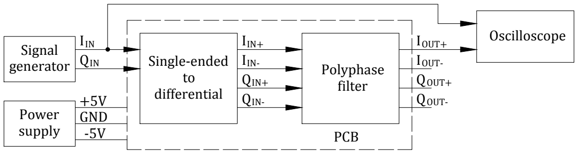


Fig. 11. Measurement setup for measurement with signal generator and oscilloscope [12].

### C. Measurement

*1) Second-order filter:* The second-order polyphase filter, with a cut-off frequency of 0.35 MHz and a frequency shift of 1.5 MHz, has an image-frequency rejection of 38.5 dB in the simulation. The measurements of the filter on the PCB at 1.5 MHz shows an image rejection of 28.7 dB. The transfer characteristic of the realized filter is slightly flatter than the simulated one (Fig. 12) [12].

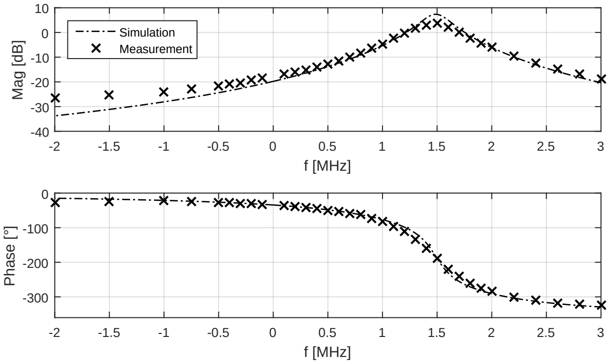


Fig. 12. Comparison of the simulation of the second-order polyphase filter and the measured values [12].

One problem is, that the filter was designed closely to the stability limit. For some measurements, the output voltage raised to +5V or -5V shortly after power up. In order to achieve a more stable behaviour, the parameters of the resistors and capacitors must be adapted. Furthermore, it is clear from the calculations of [3] that with a ratio $\frac{B}{f_S} > 1$, the value of the feedback resistor $R_{cpl2}$ goes to infinity. These resistors can be removed, which in turn simplifies the design.

*2) Fourth-order filter:* The series connection of two second-order polyphase filters yields a fourth-order filter. Measurement with the oscilloscope at $-1.5$ MHz was not performed because the amplitude of the output signal was already too small. Between the frequencies of $+1.5$ MHz and $-1.0$ MHz, a suppression of 52.7 dB was measured. The simulation shows a suppression of 70.9 dB between these frequencies [12].

## IV. CONCLUSION

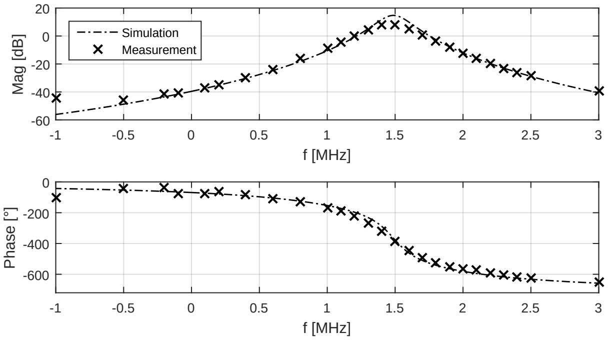


Fig. 13. Comparison of the simulation of the fourth-order polyphase filter and the measured values [12].

Polyphase filters offer the possibility to pass signals at positive frequencies and to filter the signals at the corresponding negative image frequencies, or vice versa. This enables band-limited signals to be transformed directly into the baseband with very small superimposition of the spectral components of the image frequencies. In addition, the sampling frequency can be reduced to $f_S \geq B$ using complex subsampling.

Active polyphase filters require not only passive devices but also active components such as differential amplifiers. The transfer characteristic is the same as a bandpass filter, but only for positive or negative frequencies. One can distinguish here between first and second-order filters. Second-order filters are more difficult to realize because of their more complex design, but also offer advantages. They can realize complex pole pairs and thus offer the possibility to design filters with Butterworth, Chebyshev, etc. characteristics. Due to the reduced number of cascaded stages compared to the first-order sections the power consumption is reduced too significantly.

The measured transfer characteristic of the designed filter had a somewhat flatter behaviour than in the simulation. The suppression from +1.5 MHz to -1.0 MHz was measured with 52.7 dB. The required image rejection of 55 dB could therefore not be achieved with this filter. However, this is due to the tolerances of the component parameters.

Implementing the design as an integrated circuit could further improve the filter characteristics, since in ICs, the components can be realized with significantly lower relative component tolerance. Furthermore, the entire design can be better shielded in terms of EM disturbances [12].

A design of a third-order polyphase filter is ongoing.

## ACKNOWLEDGMENT

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