Linear Phase Design of Lattice Wave Digital Filters

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***Abstract*—This paper presents the implementation of a newly developed approximation method for the design of Lattice Wave Digital Filters (LWDF) with very low group delay variations. These filters are a particular kind of Infinite Impulse Response (IIR) filters which exhibit excellent hardware implementation characteristics, since they are based on allpass filters with prescribed phase function. Allpass filters are particularly suited for hardware implementation, since the required multiplications are very low. The linear phase design method aims at minimizing the inherent group delay variations of IIR filters using modern approximation techniques from applied linear algebra. The integration into a dedicated LWDF design toolbox allows a broader audience to make use of this design method. The results show the effectiveness of this design method compared to a reference Butterworth implementation. Through the integration into the filter design toolbox it is possible to analyze quantization effects as well as to directly retrieve the filter coefficients needed for hardware implementation.**

***Index Terms*—design method, linear phase filters, allpass filters, Lattice Wave Digital Filter, LWDF, Galerkin method, Collocation method, ultra-low group delay, low multiplier count**

I. Introduction

The Lattice Wave Digital Filter is an implementation variant of the infinite impulse response filter. This type of filter generally requires less implementation effort for a given specification than a finite impulse response filter. The LWDF is a particularly favourable type due to its minimised number of multipliers and insensitivity to the effects of parameter quantisation. However, infinite impulse response filters induce a non-constant group delay to the processed signal, i.e. dispersion. Dispersion is a critical phenomenon in a variety of applications which rely on the shape of the signal envelope. The Lattice Wave Digital Filter however can be designed in a way to minimise the group delay variation. Applying a novel design method as proposed in [1], yields LWDFs with approximately constant group delay. Until now this design method has been proven effective in theory but has not yet come to practical use due to the lack of availability in commonly used filter design applications.

The intention of this work is to make this design method conveniently accessible for practical use by implementing the algorithms into a publicly available filter design toolbox [2]. Furthermore the effects of hardware implementation of LWDFs are investigated.

Like all IIR filter types LWDFs can be described using the general transfer function for a degree of N

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{m=0}^{N} b_m\, z^{-m}}{\sum_{n=0}^{N} a_n\, z^{-n}}\,, \tag{1}$$

where a and b are real coefficients. Lattice Wave Digital filters however are implemented using a distinct structure with two allpass filters $S_1(z)$ and $S_2(z)$ which are referred to in the literature [3] as reflection and reference filter respectively.

$$H(z) = \frac{1}{2}(S_1(z) + S_2(z)) \tag{2}$$

To achieve low parameter sensitivity these allpass filters are implemented in a cascaded structure of partial allpass filters of 1st and 2nd order, which are referred to as adaptors.

$$\begin{aligned} S^{(i)}(z) &= \frac{c_{i1} + z^{-1}}{1 + c_{i1}\, z^{-1}} \\ S^{(i)}(z) &= \frac{c_{i2} + c_{i1}\, z^{-1} + z^{-2}}{1 + c_{i1}\, z^{-1} + c_{i2}\, z^{-2}} \end{aligned} \tag{3}$$

where the index i represents the i-th. structure of the cascade. The transfer function of the allpass filters in cascade reads

$$S_{1,2}(z) = \prod_{i=1}^{M} S^{(i)}(z) \tag{4}$$

As can be deducted from (3), the numerator and denominator coefficients occur in pairs and reversed order. Employing an elaborate hardware implementation structure for the adaptors, the number of multipliers can be reduced approximately by a factor of two compared to the direct form implementation of (1).

The LWDF design approach generally relies on the phase relation between the two allpass filters $S_1(z), S_2(z)$. Ideally the phase relation is 0° in the passband and 180° in the stopband region, thus achieving a desired filter characteristic. Gazsi [4] proofed that Butterworth, Cauer (elliptic) and Chebychev filters of type I and II can be realized as LWDFs. LWDFs with nearly linear phase have been reported by Kunold [5] and Johansson et al [6]. In these papers one allpass filter (i.e. $S_2(z)$) is essentially a delay element with constant group delay $\tau_g = \lfloor \frac{N}{2} \rfloor$, and requires therefore no multiplication but $\lfloor \frac{N}{2} \rfloor$ delay stages (see Figure 1). The reflection filter $S_1(z)$ is designed in such a way that it approximates the phase requirements of 0° or 180° relative to the phase of $S_2(z)$. The advantage of the method is that the delay element can be realized without multipliers, the disadvantage however is a higher group delay baseline.

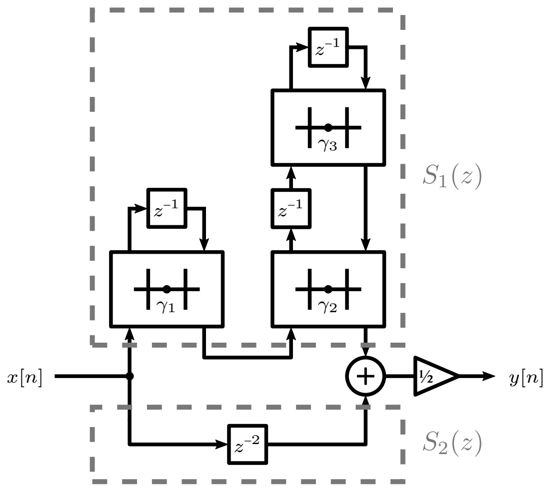

Fig. 1. Linear phase LWDF structure

II. Linear Phase Design Method

The design method featured in this paper relies on the work of Brachtendorf [1], which shows the exemplary design process for lowpass filters purely in the digital domain. In the course of the implementation however it was extended to be able to design any kind of filter characteristic through frequency transformations as described in [7]. The linear phase design method can be applied using two different approaches from applied linear algebra.

A. Galerkin approach

We again consider the rational transfer function of an infinite impulse response filter

$$H(z) = \frac{\sum_{m=0}^{N} b_m\, z^{-m}}{\sum_{n=0}^{N} a_n\, z^{-n}} \tag{5}$$

of degree N with allpass characteristic, i.e. $b_{N-n} = a_n$ holds for the coefficients of the transfer function H with $|H| = 1$. Let $\mathbf{b}$ be the vector of unknown filter coefficients of dimension $N+1$. Since we assume stability, we can replace in (5) $z = e^{j\Omega}$ obtaining the DTFT of the impulse response h.

The allpass filter shall fulfill a prescribed phase characteristic

$$\varphi(\Omega) = -\varphi(-\Omega)$$

such that

$$H\left(e^{j\Omega}\right) \approx e^{j\varphi(\Omega)}$$

with H periodic with period 2π. A re-calculation leads to the equation

$$G\left(e^{j\Omega}\right) := \sum_{n=0}^{N} b_n\, e^{-j\,n\,\Omega} - e^{j\varphi(\Omega)} \sum_{n=0}^{N} b_{N-n}\, e^{-j\,n\,\Omega} \approx 0$$

We define the inner product

$$< f,\, g >:= \frac{1}{2\pi} \int_{-\pi}^{\pi} f\, g^*\, d\Omega$$

with induced norm $\|f\|_2 = \sqrt{< f,\, f >}$, where the asterisk represents complex conjugation.

The Galerkin method calculates the coefficients by requiring that the inner product vanishes for a suitable set of test functions $\varphi_l\left(e^{j\Omega}\right)$, $l = 0, \ldots, N$, i.e.

$$< G,\, \varphi_l >= 0, \quad l = 0, \ldots, N$$

Moreover, the test functions must span an N dimensional subspace. For numerical accuracy, it is common practice to employ orthogonal basis functions $\varphi_l = e^{-j\,l\,\Omega}$, i.e. $< \varphi_l,\, \varphi_m >= \delta_{lm}$, where δ_{lm} is the Kronecker symbol. The choice of the basis functions corresponds to a Fourier series approximation.

Calculating $< G,\, \varphi_l >$, one can see from the first summation that

$$< \sum_{n=0}^{N} b_n\, e^{-j\,n\,\Omega},\, \varphi_l >= b_l$$

due to the orthogonality property. For the second summation we obtain

$$< e^{j\varphi(\Omega)} \sum_{n=0}^{N} b_{N-n}\, e^{-j\,n\,\Omega},\, \varphi_l >=$$
$$\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\varphi(\Omega)} \sum_{n=0}^{N} b_{N-n}\, e^{-j\,(n-l)\,\Omega}\, d\Omega$$

Introducing the short hand $\psi_{nl}(\Omega) = \varphi(\Omega) - (n-l)\,\Omega$ with property $\psi_{nl}(-\Omega) = -\psi_{nl}(\Omega)$, one can rewrite the integral above using trigonometric identities to

$$\frac{1}{\pi} \int_{0}^{\pi} \sum_{n=0}^{N} b_{N-n}\, \cos\psi_{nl}(\Omega)\, d\Omega := g_l(\mathbf{b}), \quad l = 0, \ldots, N$$

Defining the vector $\mathbf{g}(\mathbf{b}) = [g_0, \ldots, g_N]^T$, one obtains the linear homogeneous equation

$$\mathbf{b} - \mathbf{g}(\mathbf{b}) = 0 \tag{6}$$

i.e., the solution lies in the kernel of (6). The kernel of the homogeneous equation can be calculated, e.g., by the SVD or QR algorithm.

B. Collocation method

We define the inner product with weight function w

$$< f,\, g >_w := \frac{1}{2\pi} \int_{-\pi}^{\pi} w\, f\, g^*\, d\Omega$$

where $w(\Omega) > 0$ (up to some countable numbers in the interval $[-\pi,\, \pi[$). As test functions we employ Dirac delta distributions

$\varphi_l = \delta(\Omega - \Omega_l)$, $l = 0, \ldots, L \geq N$, $\Omega_l \in [-\pi, \pi[$. Employing the sift property of the delta distributions one obtains

$$<G, \varphi_l>_w = w(\Omega_l) \sum_{n=0}^{N} b_n \, e^{-j\, n\, \Omega_l} - w(\Omega_l)\, e^{j\varphi(\Omega_l)} \sum_{n=0}^{N} b_{N-n} \, e^{-j\, n\, \Omega_l}$$

Collecting all equations in a system of linear homogeneous equations, we get

$$[<G, \varphi_0>_w, \ldots, <G, \varphi_L>_w]^T = 0$$

For $L > N$ is the system of equations over determined. One obtains the solution with least Euclidean norm employing either the SVD or the QR algorithm. Choosing a suitable weighting function w enforces the accuracy of the approximation at specific frequencies.

III. LWDF Toolbox

The toolbox named Wave Digital Filter Designer – in which the practical part of this work is implemented – came to existence at Delft University of Technology [2]. In contrast to many existing filter design tools it focuses solely on the design of Wave Digital Filters and Lattice Wave Digital Filters hence providing an excellent basis for this work. As a plugin for Matlab it is able to take advantage of Matlab's numerical computing capabilities and the provided GUI design tool.

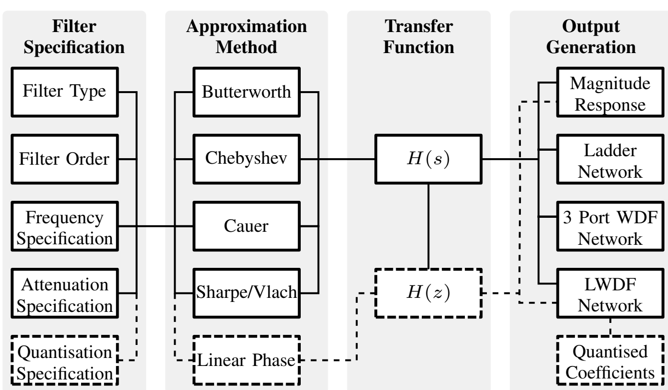

Fig. 2. Original design flow (solid) and integrated changes (dashed)

Figure 2 shows the general design flow after an initial review of the existing toolbox as well as the implemented changes to integrate the new design method. The toolbox allows the user to design filters of the four characteristic magnitude behaviours: lowpass, highpass, bandpass and bandstop. For each characteristic behaviour a number of approximation methods including Butterworth, Chebyshev, Cauer and Sharpe/Vlach can be selected. The toolbox provides the possibility to design the filters in both the continuous-time and discrete-time domains. The transformation of the two domains is done using the bilinear transformation. The results of the design process include the general transfer functions in continuous-time and discrete-time domains as well as the actual wave digital filter parameters for two and three port adaptors. For purely analog applications a ladder network containing resistors, capacitors and inductances can also be created.

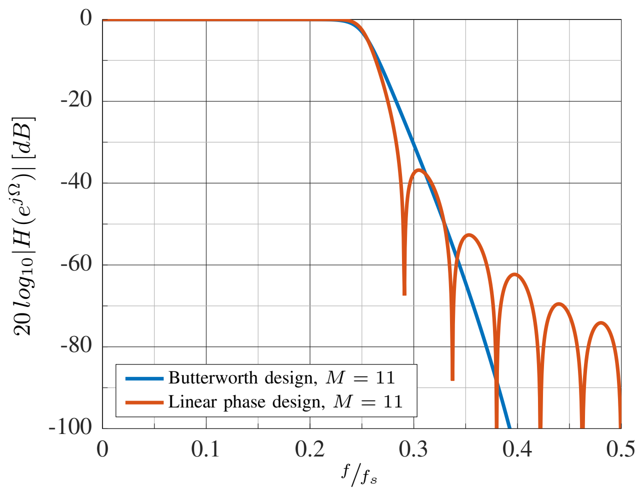

Fig. 3. Comparison of magnitude responses for the reference and linear phase design with the same implementation effort (multiplier count $M = 11$)

IV. Results

As a result of this work the linear phase approximation method was integrated into the filter design toolbox from [2] and made available for public use. The comparison of approximation methods shown in Figures 3 and 4 underlines the capabilities of the linear phase design method. As a reference for the comparison the Butterworth approximation was chosen. Since the filter order has a slightly different meaning in both approximation methods, the implementation effort (i.e. number of multipliers) of the compared filter designs is chosen to be equivalent. The defined implementation effort of $M = 11$ multipliers translates to an 11^{th} order transfer function for the reference filter and a 21^{st} order transfer function for the linear phase design. According to this relation between transfer function and filter order it can be said that the linear phase design requires only about half the multipliers of the reference design at the same order of the transfer function. The linear phase design in this example was created using the Collocation method.

As can be seen in Figure 3 the new design method features an excellent stopband behaviour with a steeper roll-off than the reference filter. With more than 70 dB stopband attenuation the linear phase design is slightly inferior to the reference design but still at a high level. The group delay plot in Figure 4 however shows the significant advantages. The group delay in the majority of the passband is approximately linear while the reference group delay gradually increases.

Adjusting the weighting function of the Collocation method, the behaviour and the emphasis of the phase correlation can be further adjusted to suit special needs. The currently used

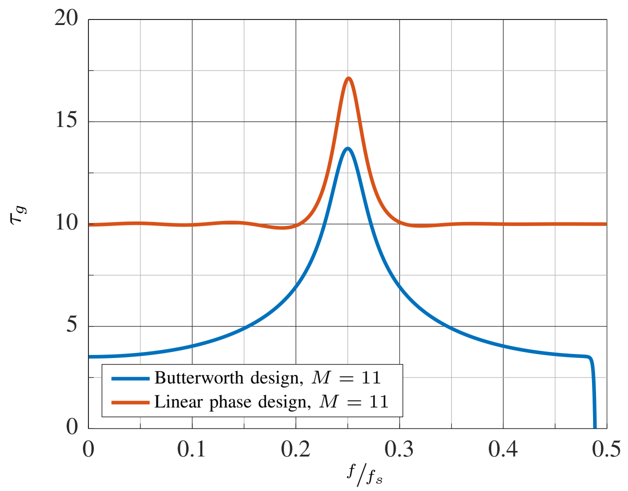

Fig. 4. Comparison of group delay variation for the reference and linear phase design with the same implementation effort (multiplier count $M = 11$)

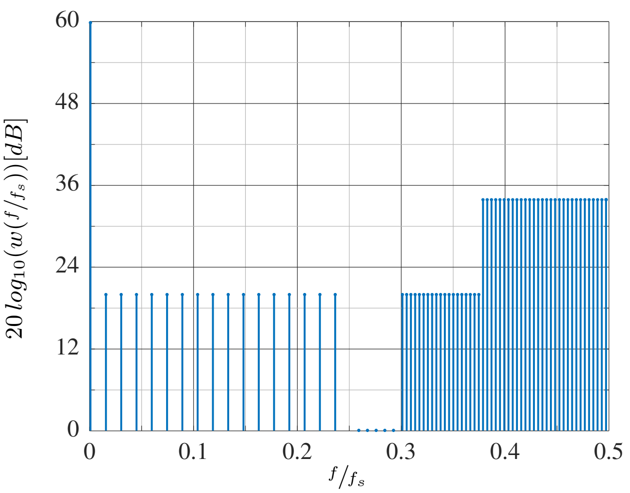

Fig. 5. Weighting function for Collocation approximation method

weighting function for the example design can be seen in Figure 5. Through spacing and magnitude of the sample points of the weighting function the optimization behaviour of the Collocation method can be adjusted.

V. Conclusion

In this document the effectiveness of the newly developed approximation method for designing Lattice Wave Digital Filters with very low group delay variations has been shown. The design procedure was made available in a filter design toolbox, using the standard LWDF topology with two parallel allpass filters in a cascaded form. It has been shown that the linear phase design requires significantly less realization effort compared to the reference design using the Butterworth approximation method while outperforming it with the exemption of the stopband attenuation. The effects of quantization of the coefficients of the transfer function can now be evaluated directly within the design procedure.

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